SPIN OBSERVABLES FOR KN SCATTERING IN MOMENTUM SPACE BASED ON A SIMPLE THREE-DIMENSIONAL BASIS

While the projectile's direction is chosen as the z direction, one can, in addition, choose the scattering plane as the xz plane, with $\hat{\mathbf{y}} = \hat{\mathbf{p}} \times \hat{\mathbf{p}}'$. Therefore, $\phi' = 0$ and

$$T_{\lambda'\lambda}(p\hat{\mathbf{p}}', p\hat{\mathbf{z}}) = T_{\lambda'\lambda}(p, p, \theta')\,, \tag{1}$$

where we have put $p' = p$. Applying Eq. (1) the spin observables for the system being considered can be derived from the following general expression [1]

$$I\,\langle\sigma^\alpha\rangle = \frac{1}{2}\sum_{\beta=0}^{3}\left\langle\sigma^\beta\right\rangle Tr\left\{T(p,p,\theta')\sigma^\beta T^+(p,p,\theta')\sigma^\alpha\right\}\,, \tag{2}$$

which connects various spin polarizations in the final and initial states, $\langle\sigma^\alpha\rangle$ and $\left\langle\sigma^\beta\right\rangle$, respectively. Here σ^0 is a 2 by 2 identity matrix describing the unpolarized condition of the projectile or that polarization of the scattered particle being not measured, while σ^α and σ^β for $\alpha, \beta = 1, 2, 3$ are components of the Pauli spin operator, refering to the following unit vectors:

$$\text{for } \sigma^\beta : \begin{cases} \hat{\mathbf{z}} &= \hat{\mathbf{p}} = \hat{\mathbf{k}}_1 \\ \hat{\mathbf{y}} &= \hat{\mathbf{p}} \times \hat{\mathbf{p}}' \\ \hat{\mathbf{x}} &= \hat{\mathbf{y}} \times \hat{\mathbf{z}} \end{cases}, \quad \text{for } \sigma^\alpha : \begin{cases} \hat{\mathbf{z}}' &= \hat{\mathbf{k}}_1' \\ \hat{\mathbf{y}}' &= \hat{\mathbf{k}}_1 \times \hat{\mathbf{k}}_1' = \hat{\mathbf{y}} \\ \hat{\mathbf{x}}' &= \hat{\mathbf{y}}' \times \hat{\mathbf{z}}' \end{cases}, \tag{3}$$

with $\mathbf{k}_1$ and $\mathbf{k}_1'$ being the initial and final momenta of the projectile, respectively, in the laboratory frame. The spin observables are listed in the following:

1. $N(K,K)N$:

$$\left.\overline{\frac{d\sigma}{d\hat{\mathbf{p}}'}}\right|_{nonrel} = (4\pi^2\mu)^2 I_0 \tag{4}$$

$$\left.\overline{\frac{d\sigma}{d\hat{\mathbf{p}}'}}\right|_{rel} = (2\pi)^4 \frac{E_1^2 E_2^2}{E^2} I_0 \tag{5}$$

$$\begin{aligned} I_0 &= \frac{1}{2} Tr\left\{T(p,p,\theta')T^+(p,p,\theta')\right\} \\ &= \left|T_{\frac{1}{2}\frac{1}{2}}(p,p,\theta')\right|^2 + \left|T_{-\frac{1}{2}\frac{1}{2}}(p,p,\theta')\right|^2 . \end{aligned} \tag{6}$$

2. $N(K,K)\vec{N}$:

$$\begin{aligned} P_y &\equiv \frac{1}{2I_0} Tr\left\{T(p,p,\theta')T^+(p,p,\theta')\sigma^y\right\} \\ &= \frac{2}{I_0} Im\left\{T^*_{\frac{1}{2}\frac{1}{2}}(p,p,\theta')T_{-\frac{1}{2}\frac{1}{2}}(p,p,\theta')\right\} . \end{aligned} \tag{7}$$

3. $\vec{N}(K,K)N$:

$$\begin{aligned} A_y \equiv & \frac{1}{2I_0} Tr\left\{T(p,p,\theta')\sigma^y T^+(p,p,\theta')\right\} \\ = & P_y\,. \end{aligned} \tag{8}$$

4. $\vec{N}(K,K)\vec{N}$:

$$\begin{aligned} D_{x'x} \equiv & \frac{1}{2I_0} Tr\left\{T(p,p,\theta')\sigma^x T^+(p,p,\theta')\sigma^{x'}\right\} \\ = & \frac{1}{I_0}\Bigg[\left\{\left|T_{\frac{1}{2}\frac{1}{2}}(p,p,\theta')\right|^2 - \left|T_{-\frac{1}{2}\frac{1}{2}}(p,p,\theta')\right|^2\right\}\cos\theta_{lab} \\ & +2Re\left\{T^*_{\frac{1}{2}\frac{1}{2}}(p,p,\theta')T_{-\frac{1}{2}\frac{1}{2}}(p,p,\theta')\right\}\sin\theta_{lab}\Bigg] \end{aligned} \tag{9}$$

$$\begin{aligned} D_{z'x} \equiv & \frac{1}{2I_0} Tr\left\{T(p,p,\theta')\sigma^x T^+(p,p,\theta')\sigma^{z'}\right\} \\ = & \frac{1}{I_0}\Bigg[\left\{\left|T_{\frac{1}{2}\frac{1}{2}}(p,p,\theta')\right|^2 - \left|T_{-\frac{1}{2}\frac{1}{2}}(p,p,\theta')\right|^2\right\}\sin\theta_{lab} \\ & -2Re\left\{T^*_{\frac{1}{2}\frac{1}{2}}(p,p,\theta')T_{-\frac{1}{2}\frac{1}{2}}(p,p,\theta')\right\}\cos\theta_{lab}\Bigg] \end{aligned} \tag{10}$$

$$\begin{aligned} D_{yy} \equiv & \frac{1}{2I_0} Tr\left\{T(p,p,\theta')\sigma^y T^+(p,p,\theta')\sigma^y\right\} \\ = & 1 \end{aligned} \tag{11}$$

$$\begin{aligned} D_{x'z} \equiv & \frac{1}{2I_0} Tr\left\{T(p,p,\theta')\sigma^z T^+(p,p,\theta')\sigma^{x'}\right\} \\ = & -D_{z'x} \end{aligned} \tag{12}$$

$$\begin{aligned} D_{z'z} \equiv & \frac{1}{2I_0} Tr\left\{T(p,p,\theta')\sigma^z T^+(p,p,\theta')\sigma^{z'}\right\} \\ = & D_{x'x}\,. \end{aligned} \tag{13}$$

The scattering angle in the laboratory frame θ_{lab} is connected to the one in the center of mass frame θ' as

$$\text{nonrel:} \quad \tan\theta_{lab} = \left(\frac{m_2}{m_1\sec\theta' + m_2}\right)\tan\theta' \tag{14}$$

$$\text{rel:} \quad \tan\theta_{lab} = \frac{M_0}{E_{lab}}\left(\frac{E_2}{E_1\sec\theta' + E_2}\right)\tan\theta'\,, \tag{15}$$

with m_1 and m_2 are the projectile's and the target's mass, respectively.

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[1] G. G. Ohlsen, Rep. Prog. Phys. **35**, 717 (1972).