Formulasi Gelombang Parsial dengan Memperhitungkan Spin

1 Basis gelombang parsial

Kita telah ketahui ekspansi keadaan basis $\big|\mathbf{p}\big\rangle$ (atau keadaan bebas dengan nomentum linier $\mathbf{p}$) dalam keadaan basis gelombang parsial $\big|plm\big\rangle$ sebagai berikut:

$$\big|\mathbf{p}\big\rangle = \sum_{lm} \big|plm\big\rangle Y^*_{lm}(\hat{\mathbf{p}})\,. \tag{1}$$

Sebaliknya, kita juga ketahui ekspansi keadaan basis gelombang parsial $\big|plm\big\rangle$ dalam keadaan basis $\big|\mathbf{p}\big\rangle$ (kebalikan dari Eq. (1)) sebagai berikut:

$$\big|plm\big\rangle = \int d\hat{\mathbf{p}} \big|\mathbf{p}\big\rangle Y_{lm}(\hat{\mathbf{p}})\,. \tag{2}$$

Kini, spin kita perhitungkan. Ambillah keadaan eigen spin, dengan sumbu z sebagai sumbu kuantisasi spin, yaitu $|s\lambda\rangle = |\hat{\mathbf{z}}s\lambda\rangle$. Keadaan basis (kita sebut juga sebagai keadaan basis 3D) di ruang momentum menjadi perkalian (*direct product*) $\big|\mathbf{p}\big\rangle$ dan $|s\lambda\rangle$, yang kita definisikan sebagai $|\mathbf{p}; s\lambda\rangle$:

$$|\mathbf{p}; s\lambda\rangle \equiv |\mathbf{p}\rangle\, |s\lambda\rangle\ . \tag{3}$$

Ortogonalitas dan relasi kekomplitan $|\mathbf{p}; s\lambda\rangle$ diberikan sebagai berikut:[1]

$$\langle \mathbf{p}'; s'\lambda'\, |\mathbf{p}; s\lambda\rangle = \delta(\mathbf{p}' - \mathbf{p})\delta_{s's}\delta_{\lambda'\lambda} \tag{6}$$

$$\sum_{s\lambda} \int d\mathbf{p}\ |\mathbf{p}; s\lambda\rangle\, \langle \mathbf{p}; s\lambda| = 1\,. \tag{7}$$

Kita masukkan Eq. (1) ke keadaan basis 3D di Eq. (3), diperoleh:

$$|\mathbf{p}; s\lambda\rangle = |\mathbf{p}\rangle\, |s\lambda\rangle = \sum_{lm} \big|plm\big\rangle Y^*_{lm}(\hat{\mathbf{p}})\, |s\lambda\rangle\ . \tag{8}$$

Momentum angular orbital $\mathbf{l}$ dan spin $\mathbf{s}$ dapat dijumlahkan menjadi momentum angular total $\mathbf{j}$:

$$\mathbf{j} = \mathbf{l} + \mathbf{s}\,. \tag{9}$$

[1] Mengingat hukun kekekalan spin, maka secara efektif yang ditemui dalam perhitungan adalah:

$$\langle \mathbf{p}'; s\lambda'\, |\mathbf{p}; s\lambda\rangle = \delta(\mathbf{p}' - \mathbf{p})\delta_{\lambda'\lambda} \tag{4}$$

$$\sum_{\lambda} \int d\mathbf{p}\ |\mathbf{p}; s\lambda\rangle\, \langle \mathbf{p}; s\lambda| = 1\,. \tag{5}$$

Dengan demikian, diperoleh:

$$\begin{aligned}
\big|\mathbf{p}; s\lambda\big\rangle &= \sum_{lm} \big|plm\big\rangle \, |s\lambda\rangle \, Y^*_{lm}(\hat{\mathbf{p}}) \\
&= \sum_{jlm} C(lsj; m\lambda) \, |p(ls)j(m+\lambda)\rangle \, Y^*_{lm}(\hat{\mathbf{p}}) \\
&= \sum_{jlm_j} C(lsj; (m_j-\lambda)\lambda) \, |p(ls)jm_j\rangle \, Y^*_{l,m_j-\lambda}(\hat{\mathbf{p}}) \, , \qquad (10)
\end{aligned}$$

dengan $|p(ls)jm_j\rangle$ adalah keadaan eigen momentum angular total, dan $C(lsj; m\lambda) = C(lsj; m\lambda(m+\lambda))$ koefisien Clebsch-Gordan (CG):

$$\mathbf{j}^2 \, |p(ls)jm_j\rangle = j(j+1) \, |p(ls)jm_j\rangle \qquad (11)$$

$$j_z \, |p(ls)jm_j\rangle = m_j \, |p(ls)jm_j\rangle \ . \qquad (12)$$

Persamaan (10) menunjukkan keadaan basis 3D $\big|\mathbf{p}; s\lambda\big\rangle$ dalam keadaan basis gelombang parsial $|p(ls)jm_j\rangle$. Kebalikan dari Eq. (10) dapat diperoleh menggunakan Eq. (2) sebagai berikut:

$$\begin{aligned}
|p(ls)jm_j\rangle &= \sum_{\lambda} C(lsj; (m_j-\lambda)\lambda) \big|pl(m_j-\lambda)\big\rangle \big|s\lambda\big\rangle \\
&= \sum_{\lambda} C(lsj; (m_j-\lambda)\lambda) \int d\hat{\mathbf{p}} \big|\mathbf{p}\big\rangle Y_{l,m_j-\lambda}(\hat{\mathbf{p}}) \big|s\lambda\big\rangle \\
&= \sum_{\lambda} C(lsj; (m_j-\lambda)\lambda) \int d\hat{\mathbf{p}} \big|\mathbf{p}; s\lambda\big\rangle Y_{l,m_j-\lambda}(\hat{\mathbf{p}}) \, . \qquad (13)
\end{aligned}$$

Perkalian skalar basis $|\mathbf{p}\rangle$ dan $|p(ls)jm_j\rangle$ diperoleh sebagai berikut:

$$\begin{aligned}
\langle \mathbf{p}'|p(ls)jm_j\rangle &= \langle \mathbf{p}'| \sum_{\lambda} C(lsj; (m_j-\lambda)\lambda) \big|pl(m_j-\lambda)\big\rangle \big|s\lambda\big\rangle \\
&= \sum_{\lambda} C(lsj; (m_j-\lambda)\lambda) \langle \mathbf{p}'\big|pl(m_j-\lambda)\big\rangle \big|s\lambda\big\rangle \\
&= \sum_{\lambda} C(lsj; (m_j-\lambda)\lambda) \frac{\delta(p'-p)}{p'p} Y_{l,m_j-\lambda}(\hat{\mathbf{p}}') \big|s\lambda\big\rangle \qquad (14)
\end{aligned}$$

$$\langle p(ls)jm_j|\mathbf{p}'\rangle = \langle \mathbf{p}'|p(ls)jm_j\rangle^* = \sum_{\lambda} C(lsj; (m_j-\lambda)\lambda) \frac{\delta(p'-p)}{p'p} Y^*_{l,m_j-\lambda}(\hat{\mathbf{p}}') \big\langle s\lambda\big| \, . \qquad (15)$$

Perkalian skalar basis $|\mathbf{p}; s\lambda\rangle$ dan $|p(ls)jm_j\rangle$ diperoleh sebagai berikut:

$$\begin{aligned}
\langle \mathbf{p}'; s'\lambda|p(ls)jm_j\rangle &= \langle \mathbf{p}'|\langle s'\lambda| \sum_{\lambda'} C(lsj; (m_j-\lambda')\lambda') \big|pl(m_j-\lambda')\big\rangle \big|s\lambda'\big\rangle \\
&= \sum_{\lambda'} C(lsj; (m_j-\lambda')\lambda') \langle \mathbf{p}'\big|pl(m_j-\lambda')\big\rangle \big\langle s'\lambda\big|s\lambda'\big\rangle
\end{aligned}$$

$$= \sum_{\lambda'} C(lsj;(m_j-\lambda')\lambda')\frac{\delta(p'-p)}{p'p}Y_{l,m_j-\lambda'}(\hat{\mathbf{p}}')\delta_{s's}\delta_{\lambda'\lambda}$$

$$= \frac{\delta(p'-p)}{p'p}\delta_{s's}C(lsj;(m_j-\lambda)\lambda)Y_{l,m_j-\lambda}(\hat{\mathbf{p}}') \tag{16}$$

$$\langle p(ls)jm_j|\mathbf{p}';s'\lambda\rangle = \langle \mathbf{p}';s'\lambda|p(ls)jm_j\rangle^* = \frac{\delta(p'-p)}{p'p}\delta_{s's}C(lsj;(m_j-\lambda)\lambda)Y^*_{l,m_j-\lambda}(\hat{\mathbf{p}}')\,. \tag{17}$$

Ortogonalitas basis gelombang parsial $|p(ls)jm_j\rangle$ diperoleh sebagai berikut:

$$\begin{aligned}
\langle p'(l's')j'm_{j'}|p(ls)jm_j\rangle &= \sum_{s''\lambda''}\int d\mathbf{p}''\,\langle p'(l's')j'm_{j'}|\mathbf{p}'';s''\lambda''\rangle\langle \mathbf{p}'';s''\lambda''|p(ls)jm_j\rangle \\
&= \sum_{s''\lambda''}\int d\mathbf{p}''\,\frac{\delta(p'-p'')}{p'p''}\delta_{s's''}C(l's'j';(m_{j'}-\lambda'')\lambda'')Y^*_{l',m_{j'}-\lambda''}(\hat{\mathbf{p}}'') \\
&\quad\times \frac{\delta(p''-p)}{p''p}\delta_{s''s}C(lsj;(m_j-\lambda'')\lambda'')Y_{l,m_j-\lambda''}(\hat{\mathbf{p}}'') \\
&= \frac{\delta(p'-p)}{p'p}\delta_{s's}\sum_{\lambda''}C(l'sj';(m_{j'}-\lambda'')\lambda'')C(lsj;(m_j-\lambda'')\lambda'') \\
&\quad\times\int d\hat{\mathbf{p}}''\,Y^*_{l',m_{j'}-\lambda''}(\hat{\mathbf{p}}'')Y_{l,m_j-\lambda''}(\hat{\mathbf{p}}'') \\
&= \frac{\delta(p'-p)}{p'p}\delta_{l'l}\delta_{s's}\delta_{m_{j'}m_j} \\
&\quad\times\sum_{\lambda''}C(lsj';(m_j-\lambda'')\lambda'')C(lsj;(m_j-\lambda'')\lambda'')\,.
\end{aligned} \tag{18}$$

Gunakan ortogonalitas koefisien CG:

$$\sum_{m_2} C(j_1j_2j_3';(m_3-m_2)m_2)C(j_1j_2j_3;(m_3-m_2)m_2) = \delta_{j_3'j_3}\,, \tag{19}$$

maka diperoleh:

$$\langle p'(l's')j'm_{j'}|p(ls)jm_j\rangle = \frac{\delta(p'-p)}{p'p}\delta_{l'l}\delta_{s's}\delta_{j'j}\delta_{m_{j'}m_j}\,. \tag{20}$$

Relasi kekomplitan basis gelombang parsial $|p(ls)jm_j\rangle$ diperoleh sebagai berikut:

$$\begin{aligned}
&\sum_{s\lambda}\int d\mathbf{p}\,|\mathbf{p};s\lambda\rangle\langle\mathbf{p};s\lambda| = 1 \\
&\to \sum_{s\lambda}\sum_{jlm_j}\sum_{j'l'm_{j'}}\int d\mathbf{p}\,C(lsj;(m_j-\lambda)\lambda)\,|p(ls)jm_j\rangle\,Y^*_{l,m_j-\lambda}(\hat{\mathbf{p}}) \\
&\qquad\qquad\times C(l'sj';(m_{j'}-\lambda)\lambda)\,\langle p(l's)j'm_{j'}|\,Y_{l',m_{j'}-\lambda}(\hat{\mathbf{p}}) = 1 \\
&\to \sum_{s\lambda}\sum_{jlm_j}\sum_{j'l'm_{j'}}\int_0^\infty dp\,p^2C(lsj;(m_j-\lambda)\lambda)C(l'sj';(m_{j'}-\lambda)\lambda)
\end{aligned}$$

$$\times\, |p(ls)jm_j\rangle\, \langle p(l's)j'm_{j'}| \int d\hat{\mathbf{p}}\, Y^*_{l,m_j-\lambda}(\hat{\mathbf{p}}) Y_{l',m_{j'}-\lambda}(\hat{\mathbf{p}}) = 1$$

$$\to \sum_{s\lambda} \sum_{jlm_j} \sum_{j'l'm_{j'}} \int_0^\infty dp\, p^2 C(lsj;(m_j-\lambda)\lambda) C(l'sj';(m_{j'}-\lambda)\lambda)$$

$$\times\, |p(ls)jm_j\rangle\, \langle p(l's)j'm_{j'}|\, \delta_{l'l}\delta_{m_{j'}m_j} = 1$$

$$\to \sum_{j'jlsm_j} \int_0^\infty dp\, p^2\, |p(ls)jm_j\rangle\, \langle p(ls)j'm_j|$$

$$\times \sum_\lambda C(lsj;(m_j-\lambda)\lambda) C(lsj';(m_j-\lambda)\lambda) = 1$$

$$\to \sum_{j'jlsm_j} \int_0^\infty dp\, p^2\, |p(ls)jm_j\rangle\, \langle p(ls)j'm_j|\, \delta_{j'j} = 1$$

$$\to \sum_{lsjm_j} \int_0^\infty dp\, p^2\, |p(ls)jm_j\rangle\, \langle p(ls)jm_j| = 1\,. \tag{21}$$

2 Persamaan T-matrix

Sifat invarian rotasional T-matrix (dan juga potensial) meminta momentum angular total tetap, begitu pula hukum kekekalan spin menuntut spin tetap:

$$\langle p'(l's')j'm_{j'}|\, T\, |p(ls)jm_j\rangle = \langle p'(l's)jm_j|\, T\, |p(ls)jm_j\rangle\, \delta_{s's}\delta_{j'j}\delta_{m_{j'}m_j}\,. \tag{22}$$

Sifat invarian rotasional juga membuat elemen matriks tidak bergantung pada orientasi kerangka acuan, pada arah sumbu koordinat, sehingga tidak bergantung pada komponen z momentum angular total, atau nilai m_j. Dengan demikian, elemen matriks T-matrix (dan elemen matriks potensial juga serupa) dalam basis gelombang parsial dinyatakan sebagai berikut, tanpa label m_j:

$$T^{js}_{l'l}(p',p) = \langle p'(l's)jm_j|\, T\, |p(ls)jm_j\rangle\ . \tag{23}$$

Persamaan untuk T-matrix dalam basis $|p(ls)jm_j\rangle$ dikerjakan sebagai berikut:

$$T = V + VG_0^{(+)}(E_p)T\ , \quad G_0^{(+)}(E_p) = \lim_{\epsilon\to 0} \frac{1}{E_p + i\epsilon - H_0} \tag{24}$$

$$\to \langle p'(l's)jm_j|T|p(ls)jm_j\rangle = \langle p'(l's)jm_j|V|p(ls)jm_j\rangle + \langle p'(l's)jm_j|VG_0^{(+)}(E_p)T|p(ls)jm_j\rangle$$

$$\to T^{js}_{l'l}(p',p) = V^{js}_{l'l}(p',p)$$

$$+ \sum_{l''s'j'm_{j'}} \int_0^\infty dp''\, p''^2 \langle p'(l's)jm_j|VG_0^{(+)}(E_p)\, |p''(l''s')j'm_{j'}\rangle\, \langle p''(l''s')j'm_{j'}|\, T|p(ls)jm_j\rangle$$

$$= V^{js}_{l'l}(p',p)$$

$$
\begin{aligned}
&+\sum_{l''s'j'm_{j'}}\int_0^\infty dp''\,p''^2\langle p'(l's)jm_j|VG_0^{(+)}(E_p)\,|p''(l''s')j'm_{j'}\rangle\,T^{js}_{l''l}(p'',p)\delta_{s's}\delta_{j'j}\delta_{m_{j'}m_j}\\
&=V^{js}_{l'l}(p',p)+\sum_{l''}\int_0^\infty dp''\,p''^2\langle p'(l's)jm_j|VG_0^{(+)}(E_p)\,|p''(l''s)jm_j\rangle\,T^{js}_{l''l}(p'',p)\\
&=V^{js}_{l'l}(p',p)\\
&\quad+\sum_{l''}\int d\mathbf{p}'''\int_0^\infty dp''\,p''^2\langle p'(l's)jm_j|VG_0^{(+)}(E_p)|\mathbf{p}'''\rangle\langle\mathbf{p}'''\,|p''(l''s)jm_j\rangle\,T^{js}_{l''l}(p'',p)\\
&=V^{js}_{l'l}(p',p)+\sum_{l''}\sum_{\lambda}C(l''sj;(m_j-\lambda)\lambda)\\
&\quad\times\int d\mathbf{p}'''\int_0^\infty dp''\,p''^2\langle p'(l's)jm_j|VG_0^{(+)}(E_p)|\mathbf{p}'''\rangle\frac{\delta(p'''-p'')}{p'''p''}Y_{l'',m_j-\lambda}(\hat{\mathbf{p}}''')\big|s\lambda\big\rangle T^{js}_{l''l}(p'',p)\\
&=V^{js}_{l'l}(p',p)+\sum_{l''}\sum_{\lambda}C(l''sj;(m_j-\lambda)\lambda)\\
&\quad\times\int d\mathbf{p}'''\,\langle p'(l's)jm_j|VG_0^{(+)}(E_p)|\mathbf{p}'''\rangle Y_{l'',m_j-\lambda}(\hat{\mathbf{p}}''')\big|s\lambda\big\rangle T^{js}_{l''l}(p''',p)\\
&=V^{js}_{l'l}(p',p)+\lim_{\epsilon\to0}\sum_{l''}\sum_{\lambda}C(l''sj;(m_j-\lambda)\lambda)\\
&\quad\times\int d\mathbf{p}'''\,\langle p'(l's)jm_j|V\frac{1}{E_p+i\epsilon-H_0}|\mathbf{p}'''\rangle Y_{l'',m_j-\lambda}(\hat{\mathbf{p}}''')\big|s\lambda\big\rangle T^{js}_{l''l}(p''',p)\\
&=V^{js}_{l'l}(p',p)+\lim_{\epsilon\to0}\sum_{l''}\sum_{\lambda}C(l''sj;(m_j-\lambda)\lambda)\\
&\quad\times\int d\mathbf{p}'''\,\langle p'(l's)jm_j|V|\mathbf{p}''';s\lambda\rangle\frac{1}{E_p+i\epsilon-E_{p'''}}Y_{l'',m_j-\lambda}(\hat{\mathbf{p}}''')T^{js}_{l''l}(p''',p)\\
&=V^{js}_{l'l}(p',p)+\lim_{\epsilon\to0}\sum_{l''}\sum_{\lambda}C(l''sj;(m_j-\lambda)\lambda)\\
&\quad\times\sum_{l'''s'j'm_{j'}}\int_0^\infty dp''\,p''^2\int d\mathbf{p}'''\,\langle p'(l's)jm_j|V\,|p''(l'''s')j'm_{j'}\rangle\,\langle p''(l'''s')j'm_{j'}|\,\mathbf{p}''';s\lambda\rangle\\
&\quad\times\frac{1}{E_p+i\epsilon-E_{p'''}}Y_{l'',m_j-\lambda}(\hat{\mathbf{p}}''')T^{js}_{l''l}(p''',p)\\
&=V^{js}_{l'l}(p',p)+\lim_{\epsilon\to0}\sum_{l''}\sum_{\lambda}C(l''sj;(m_j-\lambda)\lambda)\\
&\quad\times\sum_{l'''s'j'm_{j'}}\int_0^\infty dp''\,p''^2\int d\mathbf{p}'''\,V^{js}_{l'l'''}(p',p'')\delta_{s's}\delta_{j'j}\delta_{m_{j'}m_j}\\
&\quad\times\frac{\delta(p''-p''')}{p''p'''}C(l'''sj;(m_j-\lambda)\lambda)Y^*_{l''',m_j-\lambda}(\hat{\mathbf{p}}''')\frac{1}{E_p+i\epsilon-E_{p'''}}Y_{l'',m_j-\lambda}(\hat{\mathbf{p}}''')T^{js}_{l''l}(p''',p)\\
&=V^{js}_{l'l}(p',p)+\lim_{\epsilon\to0}\sum_{l''l'''}\sum_{\lambda}C(l''sj;(m_j-\lambda)\lambda)C(l'''sj;(m_j-\lambda)\lambda)\\
&\quad\times\int d\mathbf{p}'''\,V^{js}_{l'l'''}(p',p''')Y^*_{l''',m_j-\lambda}(\hat{\mathbf{p}}''')\frac{1}{E_p+i\epsilon-E_{p'''}}Y_{l'',m_j-\lambda}(\hat{\mathbf{p}}''')T^{js}_{l''l}(p''',p)
\end{aligned}
$$

$$
\begin{aligned}
&= V^{js}_{l'l}(p',p) + \lim_{\epsilon\to 0}\sum_{l''l'''}\sum_{\lambda} C(l''sj;(m_j-\lambda)\lambda)C(l'''sj;(m_j-\lambda)\lambda) \\
&\quad\times \int_0^\infty dp'''\, p'''^2 V^{js}_{l'l'''}(p',p''')\frac{1}{E_p+i\epsilon-E_{p'''}}T^{js}_{l'''l}(p''',p) \\
&\quad\times \int \hat{\mathbf{p}}'''\, Y^*_{l''',m_j-\lambda}(\hat{\mathbf{p}}''')Y_{l'',m_j-\lambda}(\hat{\mathbf{p}}''') \\
&= V^{js}_{l'l}(p',p) + \lim_{\epsilon\to 0}\sum_{l''}\sum_{\lambda} C(l''sj;(m_j-\lambda)\lambda)C(l''sj;(m_j-\lambda)\lambda) \\
&\quad\times \int_0^\infty dp'''\, p'''^2 V^{js}_{l'l''}(p',p''')\frac{1}{E_p+i\epsilon-E_{p'''}}T^{js}_{l'''l}(p''',p) \\
&= V^{js}_{l'l}(p',p) + \lim_{\epsilon\to 0}\sum_{l''}\int_0^\infty dp'''\, p'''^2 V^{js}_{l'l''}(p',p''')\frac{1}{E_p+i\epsilon-E_{p'''}}T^{js}_{l'''l}(p''',p) \\
\to T^{js}_{l'l}(p',p) &= V^{js}_{l'l}(p',p) + \lim_{\epsilon\to 0}\sum_{l''}\int_0^\infty dp''\, p''^2 V^{js}_{l'l''}(p',p'')\frac{1}{E_p+i\epsilon-E_{p''}}T^{js}_{l''l}(p'',p)\,. \qquad (25)
\end{aligned}
$$

Untuk menghitung penampang lintang dan besaran-besaran spin diperlukan elemen matriks T-matrix $T^s_{\lambda'\lambda}(\mathbf{p}',\mathbf{p})$, yang dapat diperoleh dari $T^{js}_{l'l}(p',p)$ sebagai berikut:

$$
\begin{aligned}
T^s_{\lambda'\lambda}(\mathbf{p}',\mathbf{p}) &= \langle \mathbf{p}';s\lambda'|T|\mathbf{p};s\lambda\rangle \\
&= \sum_{l'''l''s'''s''j'''j''m_{j'''}m_{j''}}\int_0^\infty dp'''\,dp''\,p'''^2p''^2\langle \mathbf{p}';s\lambda'\,|p'''(l'''s''')j'''m_{j'''}\rangle \\
&\quad\times \langle p'''(l'''s''')j'''m_{j'''}|\,T\,|p''(l''s'')j''m_{j''}\rangle\,\langle p''(l''s'')j''m_{j''}|\,\mathbf{p};s\lambda\rangle \\
&= \sum_{l'''l''s'''s''j'''j''m_{j'''}m_{j''}}\int_0^\infty dp'''\,dp''\,p'''^2p''^2\langle \mathbf{p}';s\lambda'\,|p'''(l'''s''')j'''m_{j'''}\rangle \\
&\quad\times T^{j''s''}_{l'''l''}(p''',p'')\delta_{s'''s''}\delta_{j'''j''}\delta_{m_{js'''}m_{j''}}\,\langle p''(l''s'')j''m_{j''}|\,\mathbf{p};s\lambda\rangle \\
&= \sum_{l'''l''s''j''m_{j''}}\int_0^\infty dp'''\,dp''\,p'''^2p''^2\langle \mathbf{p}';s\lambda'\,|p'''(l'''s'')j''m_{j''}\rangle \\
&\quad\times T^{j''s''}_{l'''l''}(p''',p'')\,\langle p''(l''s'')j''m_{j''}|\,\mathbf{p};s\lambda\rangle \\
&= \sum_{l'''l''s''j''m_{j''}}\int_0^\infty dp'''\,dp''\,p'''^2p''^2\frac{\delta(p'-p''')}{p'p'''}\delta_{s''s}C(l'''s''j'';(m_{j''}-\lambda')\lambda')Y_{l''',m_{j''}-\lambda'}(\hat{\mathbf{p}}') \\
&\quad\times T^{j''s''}_{l'''l''}(p''',p'')\frac{\delta(p''-p)}{p''p}\delta_{s''s}C(l''s''j'';(m_{j''}-\lambda)\lambda)Y^*_{l'',m_{j''}-\lambda}(\hat{\mathbf{p}}) \\
&= \sum_{l'''l''j''m_{j''}} C(l'''sj'';(m_{j''}-\lambda')\lambda')Y_{l''',m_{j''}-\lambda'}(\hat{\mathbf{p}}') \\
&\quad\times T^{j''s}_{l'''l''}(p',p)C(l''sj'';(m_{j''}-\lambda)\lambda)Y^*_{l'',m_{j''}-\lambda}(\hat{\mathbf{p}}) \\
\to T^s_{\lambda'\lambda}(\mathbf{p}',\mathbf{p}) &= \sum_{l'ljm_j} C(l'sj;(m_j-\lambda')\lambda')C(lsj;(m_j-\lambda)\lambda)Y_{l',m_j-\lambda'}(\hat{\mathbf{p}}')T^{js}_{l'l}(p',p)Y^*_{l,m_j-\lambda}(\hat{\mathbf{p}})\,. \qquad (26)
\end{aligned}
$$

Seperti biasa, ditetapkan $\hat{\mathbf{z}} = \hat{\mathbf{p}}$, sehingga diperoleh:

$$\begin{aligned}
T^s_{\lambda'\lambda}(\mathbf{p}', p\hat{\mathbf{z}}) &= \sum_{l'ljm_j} C(l'sj;(m_j-\lambda')\lambda')C(lsj;(m_j-\lambda)\lambda)Y_{l',m_j-\lambda'}(\hat{\mathbf{p}}')T^{js}_{l'l}(p',p)Y^*_{l,m_j-\lambda}(\hat{\mathbf{z}}) \\
&= \sum_{l'ljm_j} C(l'sj;(m_j-\lambda')\lambda')C(lsj;(m_j-\lambda)\lambda)Y_{l',m_j-\lambda'}(\hat{\mathbf{p}}')T^{js}_{l'l}(p',p)Y^*_{l0}(\hat{\mathbf{z}})\delta_{m_j\lambda} \\
&= \sum_{jl'l} C(l'sj;(\lambda-\lambda')\lambda')C(lsj;0\lambda)Y_{l',\lambda-\lambda'}(\hat{\mathbf{p}}')T^{js}_{l'l}(p',p)Y^*_{l0}(\hat{\mathbf{z}}) \\
&= \sum_{jl'l} \sqrt{\frac{2l+1}{4\pi}}C(l'sj;(\lambda-\lambda')\lambda')C(lsj;0\lambda)Y_{l',\lambda-\lambda'}(\hat{\mathbf{p}}')T^{js}_{l'l}(p',p)\,. \qquad (27)
\end{aligned}$$

Besaran fisika diperoleh pada keadaan on-shell, $p' = p$, sehingga: Dengan demikian, penampang lintang dihitung sebagai berikut:

$$T^s_{\lambda'\lambda}(p\hat{\mathbf{p}}', p\hat{\mathbf{z}}) = \sum_{jl'l} \sqrt{\frac{2l+1}{4\pi}}C(l'sj;(\lambda-\lambda')\lambda')C(lsj;0\lambda)Y_{l',\lambda-\lambda'}(\hat{\mathbf{p}}')T^{js}_{l'l}(p,p)\,. \qquad (28)$$