

Numerical realization for partial-wave calculation with spin

1 Analytical steps

The equation to be solved:

$$\begin{aligned}
T_{l'l}^{js}(p', p) &= V_{l'l}^{js}(p', p) + \lim_{\epsilon \rightarrow 0} \sum_{l''} \int_0^\infty dp'' \frac{p''^2}{\frac{p^2}{2\mu} + i\epsilon - \frac{p''^2}{2\mu}} V_{l'l''}^{js}(p', p'') T_{l''l}^{js}(p'', p) \\
&= V_{l'l}^{js}(p', p) + 2\mu \lim_{\epsilon \rightarrow 0} \sum_{l''} \int_0^\infty dp'' \frac{p''^2}{p^2 + i\epsilon - p''^2} V_{l'l''}^{js}(p', p'') T_{l''l}^{js}(p'', p) \\
&= V_{l'l}^{js}(p', p) + 2\mu \sum_{l''} \int_0^\infty dp'' p''^2 \left\{ \frac{\mathcal{P}}{p^2 - p''^2} - i\pi\delta(p^2 - p''^2) \right\} V_{l'l''}^{js}(p', p'') T_{l''l}^{js}(p'', p) \\
&= V_{l'l}^{js}(p', p) + 2\mu \sum_{l''} \int_0^\infty dp'' V_{l'l''}^{js}(p', p'') T_{l''l}^{js}(p'', p) \frac{p''^2}{p^2 - p''^2} \\
&\quad - i\mu\pi p \sum_{l''} V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p) \\
&\simeq V_{l'l}^{js}(p', p) + 2\mu \sum_{l''} \int_0^M dp'' V_{l'l''}^{js}(p', p'') T_{l''l}^{js}(p'', p) \frac{p''^2}{p^2 - p''^2} \\
&\quad - i\mu\pi p \sum_{l''} V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p). \tag{1}
\end{aligned}$$

Replacing the integral upper limit ∞ with a large number M is acceptable as long as the potential is a short range one. Next, taking care of the singularity.

$$\begin{aligned}
T_{l'l}^{js}(p', p) &\simeq V_{l'l}^{js}(p', p) + 2\mu \sum_{l''} \left\{ \int_0^M dp'' p''^2 V_{l'l''}^{js}(p', p'') T_{l''l}^{js}(p'', p) \frac{1}{p^2 - p''^2} \right. \\
&\quad \left. - p^2 V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p) \int_0^\infty dp'' \frac{1}{p^2 - p''^2} \right\} \\
&\quad - i\pi\mu p \sum_{l''} V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p) \\
&\simeq V_{l'l}^{js}(p', p) + 2\mu \sum_{l''} \left\{ \int_0^M dp'' p''^2 V_{l'l''}^{js}(p', p'') T_{l''l}^{js}(p'', p) \frac{1}{p^2 - p''^2} \right. \\
&\quad \left. - p^2 V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p) \int_0^M dp'' \frac{1}{p^2 - p''^2} \right. \\
&\quad \left. - p^2 V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p) \int_M^\infty dp'' \frac{1}{p^2 - p''^2} \right\}
\end{aligned}$$

$$\begin{aligned}
& -i\pi\mu p \sum_{l''} V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p) \\
& \simeq V_{l'l}^{js}(p', p) + 2\mu \sum_{l''} \int_0^M dp'' \frac{1}{p^2 - p''^2} \\
& \quad \times \left\{ p''^2 V_{l'l''}^{js}(p', p'') T_{l''l}^{js}(p'', p) - p^2 V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p) \right\} \\
& \quad - \mu p \ln \left(\frac{M-p}{M+p} \right) \sum_{l''} V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p) \\
& \quad - i\pi\mu p \sum_{l''} V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p) \\
& \simeq V_{l'l}^{js}(p', p) + 2\mu \sum_{l''} \int_0^M dp'' \frac{p''^2}{p^2 - p''^2} V_{l'l''}^{js}(p', p'') T_{l''l}^{js}(p'', p) \\
& \quad - 2\mu \sum_{l''} p^2 V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p) \int_0^M dp'' \frac{1}{p^2 - p''^2} \\
& \quad - \mu p \ln \left(\frac{M-p}{M+p} \right) \sum_{l''} V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p) \\
& \quad - i\pi\mu p \sum_{l''} V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p). \tag{2}
\end{aligned}$$

Final equation.

$$\begin{aligned}
T_{l'l}^{js}(p', p) & \simeq V_{l'l}^{js}(p', p) + 2\mu \sum_{l''} \left[\int_0^M dp'' \frac{p''^2}{p^2 - p''^2} V_{l'l''}^{js}(p', p'') T_{l''l}^{js}(p'', p) \right. \\
& \quad - \left\{ p \int_0^M dp'' \frac{1}{p^2 - p''^2} + \frac{1}{2} \ln \left(\frac{M-p}{M+p} \right) + \frac{1}{2} i\pi \right\} \\
& \quad \left. \times p V_{l'l''}^{js}(p', p) T_{l''l}^{js}(p, p) \right]. \tag{3}
\end{aligned}$$

2 Numerical steps

Discretizing the continuum variables p .

$$p' \rightarrow p_i \quad p'' \rightarrow p_k \tag{4}$$

Changing the integrals into quadratures, setting $p_{n+1} = p$, and using $\bar{\delta}_{ab} = 1 - \delta_{ab}$.

$$\begin{aligned}
T_{l'l}^{js}(p_i, p) & = V_{l'l}^{js}(p_i, p) + 2\mu \sum_{l''} \left[\sum_{k=1}^n w_k \frac{p_k^2}{p^2 - p_k^2} V_{l'l''}^{js}(p_i, p_k) T_{l''l}^{js}(p_k, p) \right. \\
& \quad - \left\{ p \sum_{r=1}^n w_r \frac{1}{p^2 - p_r^2} + \frac{1}{2} \ln \left(\frac{M-p}{M+p} \right) + \frac{1}{2} i\pi \right\} p V_{l'l''}^{js}(p_i, p_{n+1}) T_{l''l}^{js}(p_{n+1}, p) \Big] \\
& = V_{l'l}^{js}(p_i, p) + 2\mu \sum_{l''} \sum_{k=1}^{n+1} \left[\bar{\delta}_{k,n+1} \frac{w_k p_k^2}{p^2 - p_k^2} V_{l'l''}^{js}(p_i, p_k) T_{l''l}^{js}(p_k, p) \right]
\end{aligned}$$

$$\begin{aligned}
& -\delta_{k,n+1} \left\{ p \sum_{r=1}^n \frac{w_r}{p^2 - p_r^2} + \frac{1}{2} \ln \left(\frac{M-p}{M+p} \right) + \frac{1}{2} i\pi \right\} p V_{l'l''}^{js}(p_i, p_k) T_{l''l}^{js}(p_k, p) \\
& = V_{l'l}^{js}(p_i, p) \\
& + 2\mu \sum_{l''} \sum_{k=1}^{n+1} \left[\bar{\delta}_{k,n+1} \frac{w_k p_k^2}{p^2 - p_k^2} - \delta_{k,n+1} p \left\{ p \sum_{r=1}^n \frac{w_r}{p^2 - p_r^2} + \frac{1}{2} \ln \left(\frac{M-p}{M+p} \right) + \frac{1}{2} i\pi \right\} \right] \\
& \times V_{l'l''}^{js}(p_i, p_k) T_{l''l}^{js}(p_k, p) . \tag{5}
\end{aligned}$$

Bringing the 'unknowns' to the left side and constructing the system of linear equations.

$$\begin{aligned}
& T_{l'l}^{js}(p_i, p) \\
& - 2\mu \sum_{l''} \sum_{k=1}^{n+1} \left[\bar{\delta}_{k,n+1} \frac{w_k p_k^2}{p^2 - p_k^2} - \delta_{k,n+1} p \left\{ p \sum_{r=1}^n \frac{w_r}{p^2 - p_r^2} + \frac{1}{2} \ln \left(\frac{M-p}{M+p} \right) + \frac{1}{2} i\pi \right\} \right] \\
& \times V_{l'l''}^{js}(p_i, p_k) T_{l''l}^{js}(p_k, p) = V_{l'l}^{js}(p_i, p) \\
& \sum_{l''} \sum_{k=1}^{n+1} \left(\delta_{l'l''} \delta_{ik} T_{l''l}^{js}(p_k, p) \right. \\
& \left. - \left[\bar{\delta}_{k,n+1} \frac{w_k p_k^2}{p^2 - p_k^2} - \delta_{k,n+1} p \left\{ p \sum_{r=1}^n \frac{w_r}{p^2 - p_r^2} + \frac{1}{2} \ln \left(\frac{M-p}{M+p} \right) + \frac{1}{2} i\pi \right\} \right] \right. \\
& \left. \times 2\mu V_{l'l''}^{js}(p_i, p_k) T_{l''l}^{js}(p_k, p) \right) = V_{l'l}^{js}(p_i, p) \\
& \sum_{l''} \sum_{k=1}^{n+1} \left(\delta_{l'l''} \delta_{ik} \right. \\
& \left. - \left[\bar{\delta}_{k,n+1} \frac{w_k p_k^2}{p^2 - p_k^2} - \delta_{k,n+1} p \left\{ p \sum_{r=1}^n \frac{w_r}{p^2 - p_r^2} + \frac{1}{2} \ln \left(\frac{M-p}{M+p} \right) + \frac{1}{2} i\pi \right\} \right] \right. \\
& \left. \times 2\mu V_{l'l''}^{js}(p_i, p_k) \right) T_{l''l}^{js}(p_k, p) = V_{l'l}^{js}(p_i, p) . \tag{6}
\end{aligned}$$

Defining a new indexing scheme.

$$\alpha = (l'-1)(n+1) + i \quad \text{and} \quad \beta = (l''-1)(n+1) + k . \tag{7}$$

Final equation.

$$\sum_{\beta} A_{\alpha\beta}^{js}(p) T_{\beta,l}^{js}(p) = V_{\alpha,l}^{js}(p) , \tag{8}$$

with p being the energy parameter and

$$\begin{aligned}
A_{\alpha\beta}^{js}(p) & = \delta_{l'l''} \delta_{ik} - \left[\bar{\delta}_{k,n+1} \frac{w_k p_k^2}{p^2 - p_k^2} - \delta_{k,n+1} p \left\{ p \sum_{r=1}^n \frac{w_r}{p^2 - p_r^2} + \frac{1}{2} \ln \left(\frac{M-p}{M+p} \right) + \frac{1}{2} i\pi \right\} \right] \\
& \times 2\mu V_{l'l''}^{js}(p_i, p_k) \tag{9}
\end{aligned}$$

$$T_{\beta,l}^{js}(p) = T_{l''l}^{js}(p_k, p) \tag{10}$$

$$V_{\alpha,l}^{js}(p) = V_{l'l}^{js}(p_i, p) . \tag{11}$$