Metode Numerik

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Isi

PD Orde 2

Bentuk umum PD orde 2: $y'' = \frac{d^2y}{dx^2} = f(x, y, y')$

Diketahui: $y(x_0) = y_0, \quad y'(x_0) = y'_0 \longrightarrow y(x) = ?$

Definisikan fungsi baru u:

$$\begin{aligned} u &= y' \\ u_0 &= y'_0 \end{aligned} \longrightarrow \begin{aligned} y' &= u(x, y) \\ u' &= f(x, y, u) \end{aligned}$$

Masalah PD orde 2
berubah menjadi
masalah PD orde 1.

Contoh penyelesaian dengan metode Euler yang lebih baik (improved):

$u' = f(x,y,u)$	$y' = u(x,y)$
$u(x_0+h) = u_0 + \tfrac{1}{2}h(f_0 + f_1)$	$y(x_0+h) = y_0 + \tfrac{1}{2}h(u_0 + u_1)$
$f_0 = f(x_0, y_0, u_0)$	$u_0 = y'_0$
$f_1 = f(x_0+h, y_0+hu_0, u_1)$	$u_1 = u_0 + hf_0$

Alur perhitungan:

$$
\begin{tikzcd}
y_0,\ u_0 \arrow[r] & f_0 \arrow[r] & u_1 \arrow[r] & f_1,\ y(x_0+h),\ u(x_0+h) \arrow[d] \\
 & x_0+h \to x_0,\ u(x_0+h) \to u_0,\ y(x_0+h) \to y_0 \arrow[ul] & & \arrow[ll]
\end{tikzcd}
$$

Contoh penyelesaian dengan metode Runge-Kutta orde 4:

$$u' = f(x, y, u) \qquad\qquad y' = u(x, y)$$

$$u(x_0 + h) = u_0 + \tfrac{1}{6}h\left(f_0 + 2f_1 + 2f_2 + f_3\right) \qquad y(x_0 + h) = y_0 + \tfrac{1}{6}h\left(u_0 + 2u_1 + 2u_2 + u_3\right)$$

$$f_0 = f(x_0, y_0, u_0) \qquad u_0 = y'_0$$

$$f_1 = f(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}hu_0, u_1) \qquad u_1 = u_0 + \tfrac{1}{2}hf_0$$

$$f_2 = f(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}hu_1, u_2) \qquad u_2 = u_0 + \tfrac{1}{2}hf_1$$

$$f_3 = f(x_0 + h, y_0 + hu_2, u_3) \qquad u_3 = u_0 + hf_2$$

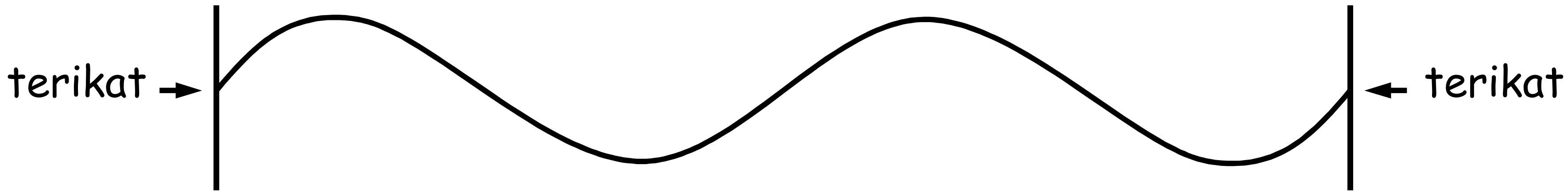
Alur perhitungan:

$$y_0,\ u_0 \longrightarrow f_0 \longrightarrow u_1 \longrightarrow f_1 \longrightarrow u_2 \longrightarrow f_2 \longrightarrow u_3 \longrightarrow f_3,\ y(x_0 + h),\ u(x_0 + h)$$

$$x_0 + h \rightarrow x_0,\ u(x_0 + h) \rightarrow u_0,\ y(x_0 + h) \rightarrow y_0$$

PD dengan Syarat Batas

Contoh, gelombang yang merambat di sepanjang tali bisa digambarkan dengan PD orde 2. Jika ujung-ujung tali itu diikat sehingga tidak bisa bergerak, maka kita temui kasus PD dengan syarat batas.



Bentuk umum PD orde 2 linear: $y'' = g(x, y, y') = a(x) - b(x)y - c(x)y'$

Diketahui:

$$x_0 \le x \le x_n$$
$$y(x_0) = y_0 \longrightarrow y(x) = ?$$
$$y(x_n) = y_n$$

Dicari $y_i = y(x_i)$ pada titik $x_i = x_0 + ih \ (i = 1, \ldots, n-1)$ dengan $h = \dfrac{x_n - x_0}{n}$.

Metode Finite Differences

$$y''+c(x)y'+b(x)y=a(x)$$

$$y'_i \cong \frac{y_{i+1}-y_{i-1}}{2h}$$

$$y''_i \cong \frac{y_{i+1}-2y_i+y_{i-1}}{h^2}$$

$$y''_i+c_iy'_i+b_iy_i=a_i$$

$$\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}+c_i\frac{y_{i+1}-y_{i-1}}{2h}+b_iy_i \cong a_i$$

Jadi, pada akhirnya ditemui masalah sistem persamaan linear:

$$\boxed{\left(1-\frac{c_ih}{2}\right)y_{i-1}-\left(2-b_ih^2\right)y_i+\left(1+\frac{c_ih}{2}\right)y_{i+1} \cong a_ih^2}$$

yang dapat diselesaikan menggunakan metode, contoh, iterasi Jacobi dan Gauss-Siedel.

Aplikasi Iterasi Jacobi dan Gauss-Siedel pada PD dengan Syarat Batas

PD orde 2: $y'' = a(x) - b(x)y - c(x)y' \rightarrow \left(1 - \frac{c_i h}{2}\right) y_{i-1} - \left(2 - b_i h^2\right) y_i + \left(1 + \frac{c_i h}{2}\right) y_{i+1} \cong a_i h^2$

Iterasi Jacobi:

$$\boxed{y_i^{(k)} \cong \frac{1}{\left(2 - b_i h^2\right)} \left(-a_i h^2 + \left(1 - \frac{c_i h}{2}\right) y_{i-1}^{(k-1)} + \left(1 + \frac{c_i h}{2}\right) y_{i+1}^{(k-1)} \right)}$$

Iterasi Gauss-Siedel (contoh untuk i membesar, i = 1, …, n-1):

$$\boxed{y_i^{(k)} \cong \frac{1}{\left(2 - b_i h^2\right)} \left(-a_i h^2 + \left(1 - \frac{c_i h}{2}\right) y_{i-1}^{(k)} + \left(1 + \frac{c_i h}{2}\right) y_{i+1}^{(k-1)} \right)}$$

Catatan, sesuai syarat batas: $\boxed{y_0^{(k)} = y_0 \qquad y_n^{(k)} = y_n}$