Kinematika Hamburan Nonrelativistik

- relasi momentum $\mathbf{k}$ dan energi kinetik E:

$$E = \frac{\mathbf{k}^2}{2m} \tag{1}$$

- kecepatan $\mathbf{v}$:

$$\mathbf{v} = \frac{\mathbf{k}}{m} \tag{2}$$

- dalam kerangka acuan laboratorium, partikel 1 sebagai proyektil dan partikel 2 sebagai target **diam**, berlaku hukum kekekalan energi:

$$\begin{aligned} E_{lab} &= E'_{lab} \\ \rightarrow E_{1,lab} &= E'_{1,lab} + E'_{2,lab} \\ \rightarrow \frac{\mathbf{k}_1^2}{2m_1} &= \frac{\mathbf{k}_1'^2}{2m_1} + \frac{\mathbf{k}_2'^2}{2m_2} \end{aligned} \tag{3}$$

dan hukum momentum linier:

$$\begin{aligned} \mathbf{K} &= \mathbf{K}' \\ \rightarrow \mathbf{k}_1 &= \mathbf{k}'_1 + \mathbf{k}'_2 \end{aligned} \tag{4}$$

- dalam kerangka acuan pusat massa, gerakan pusat massa tidak ada, secara efektif yang diamati adalah gerakan satu benda dengan massa tereduksi μ dan momentum linier awal $\mathbf{p}$ dan akhir $\mathbf{p}'$energi tetap:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \tag{5}$$

$$\mathbf{p}' = p\hat{\mathbf{p}}' \tag{6}$$

dan berlaku hukum kekekalan energi:

$$\begin{aligned} E &= E' \\ \rightarrow \frac{\mathbf{p}^2}{2\mu} &= \frac{\mathbf{p}'^2}{2\mu} \end{aligned} \tag{7}$$

- relasi momentum dalam kerangka laboratorium dan momentum dalam kerangka pusat massa:

$$\mathbf{p}' = \frac{m_2\mathbf{k}'_1 - m_1\mathbf{k}'_2}{m_1 + m_2} \tag{8}$$

$$\mathbf{p} = \frac{m_2\mathbf{k}_1}{m_1 + m_2} = \frac{\mu}{m_1}\mathbf{k}_1 \tag{9}$$

- relasi energi dalam kerangka laboratorium dan energi dalam kerangka pusat massa:

$$
\begin{aligned}
E_{lab} &= E_{lab}^{(pusat\ massa)} + E \\
\rightarrow \frac{\mathbf{k}_1^2}{2m_1} &= \frac{\mathbf{K}^2}{2(m_1+m_2)} + \frac{\mathbf{p}^2}{2\mu} = \frac{\mathbf{k}_1^2}{2(m_1+m_2)} + \frac{\mathbf{p}^2}{2\mu}
\end{aligned} \tag{10}
$$

$$
E = \frac{\mathbf{p}^2}{2\mu} = \frac{1}{2\mu}\frac{\mu^2}{m_1^2}\mathbf{k}_1^2 = \frac{\mu}{m_1}E_{lab} \tag{11}
$$

- Dari persamaan (11) dan (7), disimpulkan bahwa E_{lab} menjadi input bagi perhitungan hamburan: $E_{lab} \rightarrow E = \frac{\mu}{m_1}E_{lab}$, kemudian $E \rightarrow p = \sqrt{2\mu E}$.

- relasi sudut hambur dalam kerangka laboratorium θ_{lab} dan sudut hambur dalam kerangka pusat massa θ:

$$
\cos\theta_{lab} = \hat{\mathbf{k}}_1' \cdot \hat{\mathbf{k}}_1 = \hat{\mathbf{k}}_1' \cdot \hat{\mathbf{p}} \tag{12}
$$

$$
\cos\theta = \hat{\mathbf{p}}' \cdot \hat{\mathbf{p}} = \hat{\mathbf{p}}' \cdot \hat{\mathbf{k}}_1 \tag{13}
$$

$$
\begin{aligned}
\mathbf{p}' &= \frac{m_2\mathbf{k}_1' - m_1\mathbf{k}_2'}{m_1+m_2} \\
&= \frac{m_2\mathbf{k}_1' - m_1(\mathbf{k}_1 - \mathbf{k}_1')}{m_1+m_2} \\
&= \mathbf{k}_1' - \frac{m_1}{m_1+m_2}\mathbf{k}_1 \\
&= \mathbf{k}_1' - \frac{\mu}{m_2}\mathbf{k}_1 \\
&= \mathbf{k}_1' - \frac{m_1}{m_2}\mathbf{p} \\
\rightarrow \mathbf{k}_1' &= \mathbf{p}' + \frac{m_1}{m_2}\mathbf{p}
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
k_1'^2 &= p'^2 + \frac{m_1^2}{m_2^2}p^2 + 2\frac{m_1}{m_2}\mathbf{p}' \cdot \mathbf{p} \\
&= p'^2 + \frac{m_1^2}{m_2^2}p^2 + 2\frac{m_1}{m_2}p'p\cos\theta \\
&= p^2\left(1 + \frac{m_1^2}{m_2^2} + 2\frac{m_1}{m_2}\cos\theta\right)
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
k_1\cos\theta_{lab} &= \mathbf{k}_1' \cdot \hat{\mathbf{p}} \\
&= \mathbf{p}' \cdot \hat{\mathbf{p}} + \frac{m_1}{m_2}\mathbf{p} \cdot \hat{\mathbf{p}} \\
&= p\cos\theta + \frac{m_1}{m_2}p \\
&= p\left(\cos\theta + \frac{m_1}{m_2}\right)
\end{aligned} \tag{16}
$$

$$k_1^2 \cos^2\theta_{lab} = p^2\left(\cos^2\theta + \frac{m_1^2}{m_2^2} + 2\frac{m_1}{m_2}\cos\theta\right) \tag{17}$$

$$\begin{aligned} k_1^2 \sin^2\theta_{lab} &= k_1^2 - k_1^2\cos^2\theta_{lab} \\ &= p^2\left(1-\cos^2\theta\right) \\ &= p^2\sin^2\theta \end{aligned} \tag{18}$$

$$k_1 \sin\theta_{lab} = p\sin\theta \tag{19}$$

$$\begin{aligned} \rightarrow \tan\theta_{lab} &= \frac{k_1\sin\theta_{lab}}{k_1\cos\theta_{lab}} \\ &= \frac{p\sin\theta}{p\left(\cos\theta + \frac{m_1}{m_2}\right)} \\ &= \frac{\sin\theta}{\cos\theta + \frac{m_1}{m_2}} \\ &= \left(\frac{1}{1+\frac{m_1}{m_2}\sec\theta}\right)\tan\theta \\ &= \left(\frac{m_2}{m_2+m_1\sec\theta}\right)\tan\theta \end{aligned} \tag{20}$$

- khusus untuk $m_1 = m_2 = m$ diperoleh:

$$\mu = \frac{m^2}{2m} = \frac{1}{2}m \tag{21}$$

$$\mathbf{p} = \frac{m\mathbf{k}_1}{2m} = \frac{1}{2}\mathbf{k}_1 \tag{22}$$

$$\mathbf{p}' = \frac{m\mathbf{k}_1' - m\mathbf{k}_2'}{2m} = \frac{1}{2}(\mathbf{k}_1' - \mathbf{k}_2') \tag{23}$$

$$E = \frac{m}{2m}E_{lab} = \frac{1}{2}E_{lab} \tag{24}$$

$$\begin{aligned} \tan\theta_{lab} &= \left(\frac{1}{1+\sec\theta}\right)\tan\theta \\ &= \left(\frac{\cos\theta}{\cos\theta+1}\right)\tan\theta \\ &= \frac{\sin\theta}{\cos\theta+1} \\ &= \frac{2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}{2\cos^2\frac{1}{2}\theta} \\ &= \tan\frac{1}{2}\theta \end{aligned}$$

$$\rightarrow \theta = 2\theta_{lab} \tag{25}$$