foil. The prevalent model for an atom at the time was J. J. Thomson's "watermelon" model, in which negative electrons floated in a ball of positive charge. The relatively large angle suffered by a small fraction of the α particles in the incident beam in Rutherford's experiment was found to be inconsistent with Thomson's model of the atom. For it is easily shown that α particles, after passing through hundreds of such spheres of distributed charge, are deflected at most only by a few degrees. On the other hand the actual scattering data is consistent with an atomic model in which the positive charge is concentrated in a central core of small diameter. Large angle of scatter is then experienced by α particles which pass sufficiently close to the positive nucleus.

Scattering Cross Section

The typical configuration of a scattering experiment is shown in Fig. 14.1. A uniform monoenergetic beam of particles of known energy and current density $\mathbf{J}_{\text{inc}}$ (7.107) is incident on a target containing scattering centers. Such scattering centers might, for example, be the positive nuclei of atoms in a metal lattice. If the particles in the incident beam are, say, α particles, then when one such particle comes sufficiently close to one of the nuclei in the sample, it will be scattered. If the target sample is sufficiently thin, the probability of more than one such event for any particle in the incident beam is small and one may expect to obtain a valid description of the scattering data in terms of a single two-particle scattering event.

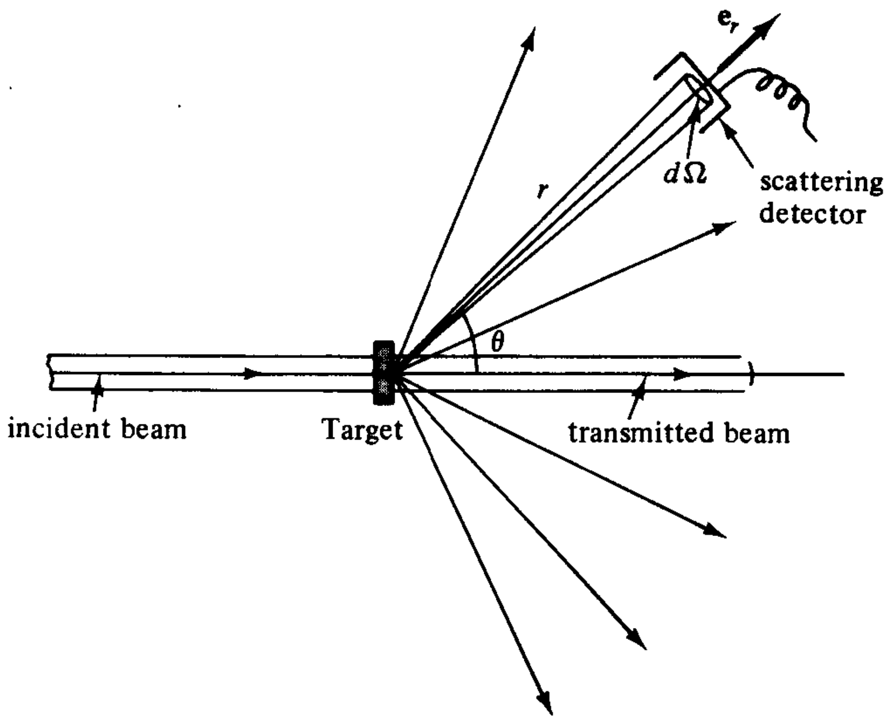

FIGURE 14.1 Scattering configuration.

Let the scattered current be $\mathbf{J}_{sc}$. Then the number of particles per unit time scattered through some surface element $d\mathbf{S}$ is $\mathbf{J}_{sc} \cdot d\mathbf{S}$. Let $d\mathbf{S}$ be at the radius $\mathbf{r}$ from the target. Then if $d\boldsymbol{\Omega}$ is the vector solid angle subtended by $d\mathbf{S}$ about the target origin, $d\mathbf{S} = r^2\, d\boldsymbol{\Omega}$ (see Figs. 9.9 and 14.1). The vector solid angle $d\boldsymbol{\Omega}$ is in the direction of $\mathbf{e}_r$; that is, $d\boldsymbol{\Omega} = \mathbf{e}_r\, d\Omega$. It follows that

$$\begin{array}{l}\text{number of particles}\\ \text{passing through } d\mathbf{S} = dN = \mathbf{J}_{sc} \cdot d\mathbf{S} = r^2 \mathbf{J}_{sc} \cdot d\boldsymbol{\Omega}\\ \text{per second}\end{array}$$

Since the number of such scattered particles will grow with the incident current $\mathbf{J}_{inc}$, one may assume this number to be proportional to $\mathbf{J}_{inc}$ and can equate

$$dN = r^2 \mathbf{J}_{sc} \cdot d\boldsymbol{\Omega} = \mathbf{J}_{inc}\, d\sigma \tag{14.1}$$

The proportionality factor $d\sigma$ is called the *differential scattering cross section* and has dimensions of cm^2. It may be interpreted as an obstructional area which the scatterer presents to the incident beam. Particles taken out of the incident beam by this obstructional area are scattered into $d\boldsymbol{\Omega}$. The *total scattering cross section* σ represents the obstructional area of scattering in all directions.

$$\sigma = \int d\sigma = \int_{4\pi} \left(\frac{d\sigma}{d\Omega}\right) d\Omega \tag{14.2}$$

Scattering cross section has a classical counterpart. Classically, the total cross section seen by a uniform beam of point particles incident on a fixed rigid sphere of radius a is $\sigma = \pi a^2$. If the incident beam has current $\mathbb{J}_{inc}$, the number per second scattered out of the beam in all directions is $\pi a^2 \mathbb{J}_{inc}$.

The Scattering Amplitude

Returning to quantum mechanics, let the particles in the incident beam be independent of each other so that prior to interaction with the target a particle in the incident beam may be considered a *free particle*. If the z axis is taken to coincide with the axis of incidence, then a particle in the incident beam with momentum $\hbar\mathbf{k}$ and energy $\hbar^2 k^2/2m$ is in the planewave state,

$$\varphi_{inc} = e^{ikz} \tag{14.3}$$

When this wave interacts with a scattering center, an outgoing scattered wave φ_{sc} is initiated. If the scattering is *isotropic* so that scattering into all directions (all 4π steradians of solid angle) is equally probable, we can expect the scattered wave φ_{sc} to